Teorema da Função Implícita.

Conjuntos definidos por equações.

$\boxed{\text{Variedades}}$

$$F : \mathbb{R}^n \to \mathbb{R}^m, \quad m < n, \quad C^1$$

$$\mathbb{R}^n = \mathbb{R}^{n-m} \times \mathbb{R}^{m}$$

(x, y)

$\boxed{\begin{array}{l} n=2 \\ m=1 \end{array}}$ $\boxed{\begin{array}{l} n=3 \\ m=1 \end{array}}$ $\boxed{\begin{array}{l} n=3 \\ m=2 \end{array}}$

$m \times m$

$$\underbrace{DF(x,y)}_{m \times n} = \Big[D_x F \;\Big|\; \underbrace{\boxed{D_y F}}_{\det \neq 0} \Big]$$

$$\boxed{F(x,y) = 0} \iff y = f(x), \; f \in C^1$$

localmente existe, C^1

$\boxed{\text{implícita}}$

$$M = \{(x,y) \in \mathbb{R}^{n-m} \times \mathbb{R}^m : F(x,y) = 0\}$$

$$DF(x,y) = \left[\quad ; D_y F(x,y) \right]_{m \times n}$$

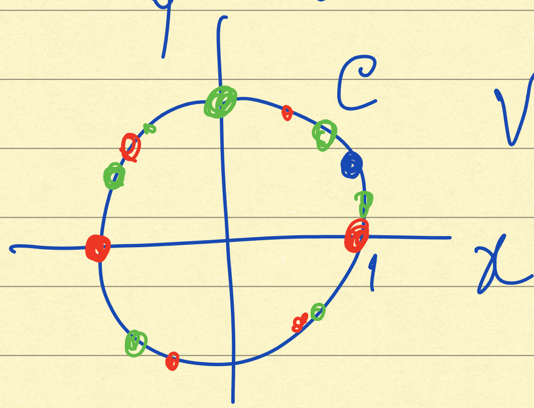
$\forall (x,y) \in M$ $\quad$ $\det \neq 0$

diz-se que M é uma variedade de dimensão $(n-m)$.

$(n-m) \equiv$ nº de variáveis livres

$$\boxed{M \equiv \text{variedade}_{(n-m)}}$$

Exemplos 1) $C = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$



Variedade-1

$\dim(C) = 1$

$F(x,y) = x^2 + y^2 - 1 = 0$

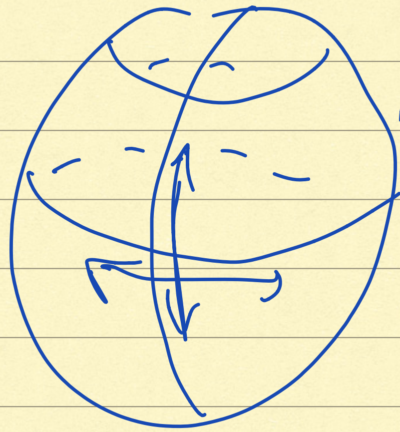
$F : \mathbb{R}^2 \to \mathbb{R}, \ C^1$

$DF(x,y) = [\,2x \quad 2y\,] \neq [\,0 \quad 0\,]$

$\neq 0 \quad \neq 0 \ \forall (x,y) \in C$

$x = x(y)$ $\qquad$ $y = y(x)$

2) $M = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2=1\}$



$\dim(M)=2$ //

$F(x,y,z) = x^2+y^2+z^2-1 = 0$

$F: \mathbb{R}^3 \to \mathbb{R}, \quad C^1$

$DF(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \end{bmatrix}$

$\neq \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

$\forall (x,y,z) \in M$

$M \equiv$ variedade-2 //

3) $N = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1 ;$
$x = y \}$

$$F(x,y,z) = (x^2+y^2+z^2-1,\ x-y) = (0,0)$$

$F : \mathbb{R}^3 \to \mathbb{R}^2$, C^1

$$DF(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \\ 1 & -1 & 0 \end{bmatrix}$$

$\det \neq 0$

Pelo menos um dos blocos 2x2 tem determinante diferente de zero.

$$\begin{cases} y = y(x) \\ z = z(x) \end{cases}, \quad C^1 \qquad x \text{ livre}$$

N = Variedade - 1

$$\begin{bmatrix} 2x & 2y & 2z \\ 1 & -1 & 0 \end{bmatrix}$$

Pelo menos um dos blocos 2x2 tem determinante $\neq 0$.

Suponhamos que não:

$$\begin{cases} -2x-2y=0 \\ 2z=0 \\ -2z=0 \end{cases} \qquad \begin{cases} y=-x \\ z=0 \end{cases}$$

Mas $\begin{cases} x^2+y^2+z^2=1 \\ y=x \end{cases}$ $\qquad \begin{cases} x^2+y^2=1 \\ y=x \\ y=-x \end{cases}$ $\quad y=x=0$

$$\begin{cases} x^2+y^2=1 \\ y=x=0 \end{cases} \qquad \begin{cases} 0=1 \end{cases}$$

$\boxed{\text{absurdo}}$

$F: \mathbb{R}^n \longrightarrow \mathbb{R}^m$, $m<n$, C^1

$$DF(\cdot) = \left[\quad \vdots \underset{m\times m}{\det \neq 0} \right]_{m \times n}$$

m = nº de var. dependentes

$n-m$ = dimensão = nº de var. livres.

$$\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

linhas são vetores linear/independentes.

se

$$M = \{ x \in \mathbb{R}^n : F(x) = 0 \}$$

$DF(x)$ tem linhas lin. ind.

$\forall x \in M$, então M = variedade-$(n-m)$

linhas de $DF(\cdot)$ linear/ ind.

$\Leftrightarrow$ característica de $DF(\cdot)$ é máxima $= m$

$$DF(\cdot) = \begin{bmatrix} - & - & - & | & - & \dots \\ - & - & - & | & \underline{m \times m} & \\ - & - & & | & - & \end{bmatrix}_{m \times n}$$

$F: \mathbb{R}^n \longrightarrow \mathbb{R}^m$, $m < n$, C^1

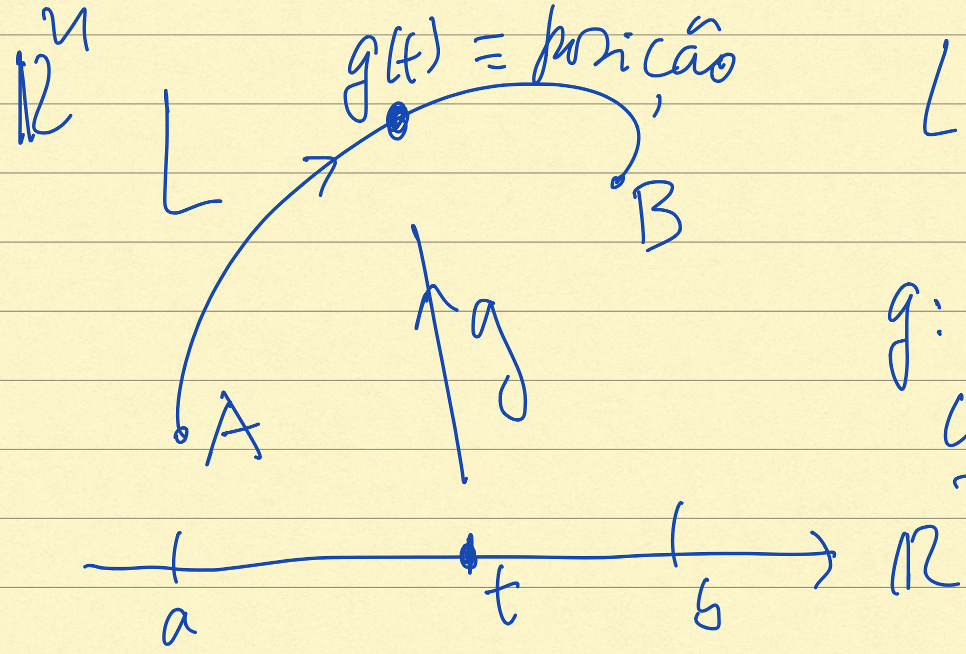
$F = (F_1, F_2, \dots, F_m)$; $F_j : \mathbb{R}^n \to \mathbb{R}$

$$DF(\cdot) = \begin{array}{l} \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_1}{\partial x_n} \\ - & - & \dots \\ \frac{\partial F_m}{\partial x_1} & \dots & \frac{\partial F_m}{\partial x_n} \end{bmatrix}_{m \times n} \end{array} \begin{array}{l} \to DF_1 \\ \\ \to DF_m \end{array}$$

$$DF(\cdot) = \begin{bmatrix} \cdots & DF_1 & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & DF_m & \cdots \end{bmatrix}$$

$$M = \left\{ x \in \mathbb{R}^n : \begin{cases} F_1(x) = 0 \\ \vdots \\ F_m(x) = 0 \end{cases} \right\}$$

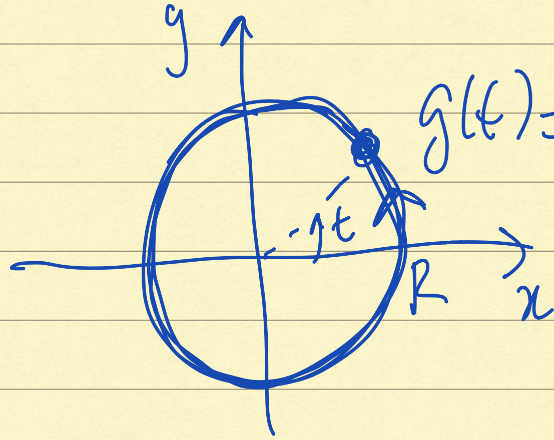
Recordar: linha em $\mathbb{R}^n$.



L = imagem de g

$g : \mathbb{R} \to \mathbb{R}^n$ contínua.

1)

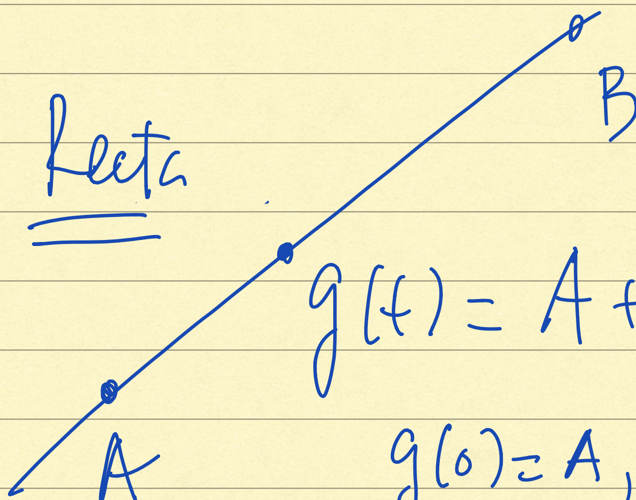


$g(t) = (R\cos t, R\,\mathrm{sen}\, t)$

$0 \leq t \leq 2\pi$

Circunferência

2) Recta



$g(t) = A + t(B - A), \; t \in \mathbb{R}$

$g(0) = A, \quad g(1) = B$